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FLEXURAL WRINKLING OF HONEYCOMB SANDWICH BEAMS WITH LAMINATED FACES

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Abstract—A theory is presented for the wrinkling of honeycomb sandwich beams, which satisfies the governing elasticity equations for the orthotropic core and laminated faces. A single eighth order ordinary differential equation is obtained which is solved by assuming a simple sine function for the displacement of the buckled face. Minimum critical loads are found numerically, and results are given for typical beams which show the dependence on face plate thickness and core thickness. A comparison is made with other approximate solutions.

NOTATION

A	in-plane stiffness
a_n	constants
B	coupling stiffness term
b	half-width of beam
b_n	constants
C	core modulus in compression
D	flexural stiffness term
E	Young's modulus
F	force in face plate per unit width
h	core thickness between face plate mid-planes
K_1, K_2, K_3	functions arising from integrating core equations
L_{xz}	core shear modulus
l	beam length
M_x	moment in face plate per unit width
N_x	force in face plate per unit width
n	number of half waves
$q_1 \ldots q_5$	defined below eqn (20)
t	face plate thickness
u	displacement in x direction
w	displacement in z direction
x, z	axial and normal co-ordinate axes
γ_{xz}	core shear strain
ϵ_x, ϵ_z	direct strains
κ_x	face plate curvature
σ_x, σ_z	direct stresses
τ_{xz}	core shear stress

Subscripts

0	prior to wrinkling
1	after wrinkling

Superscript

F	face plate variables

1. INTRODUCTION

One possible mode of failure of a thin faced sandwich beam subjected to bending is the localized buckling (wrinkling) of the compression face. This paper presents a small deflection analysis of the problem, where the sandwich beam has a specially orthotropic core and laminated cross-ply faces. The governing elasticity equations are considered separately for the core and faces and then combined through the displacement boundary conditions at the common inter-face. They are manipulated to form a single governing differential equation which is then solved by assuming a sinusoidal variation for the normal displacement of the face during wrinkling. Numerical results are obtained for beams having typical carbon fibre faces and honeycomb cores, which show the variation of critical load with face plate thickness and core thickness. Comparisons are made with other approximate solutions in the literature for isotropic faces and cores.

The work is an extension of the theory given in Ref. [1] for the buckling of columns and has important practical applications. For example, a helicopter rotor blade may be manufactured with a full depth honeycomb core and one of the design constraints could be the possibility of wrinkling of the compression face when the blade is subjected to bending. Furthermore, modern blade designs use unidirectional glass fibre reinforced plastic as the main bending material with a ±45° fibre weave to take the torsional loads so that it is important to take into account the laminated nature of the faces.

2. THEORY

Figure 1 shows part of a sandwich beam in which the effect of a uniform bending moment is represented by a compressive force F per unit width on the top face and a tension force F on the bottom face. The honeycomb core is specially orthotropic with zero direct stiffness in the x direction so that it does not contribute to reacting the bending moment and is assumed to give continuous support to the faces. The problem may be considered as a two dimensional one in generalised plane stress in the same way as the sandwich column under axial compression[1]. The governing equations may be considered in two parts; those prior to wrinkling and those after wrinkling. Prior to wrinkling it will be assumed that the faces are simply compressed and extended with displacements $u_0{}^F$. This implies that they are held straight by the core and that therefore $w_0{}^F = 0$. It will be further assumed that the core stresses induced during this initial phase are small and can be neglected.

We shall take the faces to be of laminated construction, with each individual ply being specially orthotropic with respect to the x axis. The full set of stress (load)-strain equations can be obtained from Ref. [2], but they are reduced, as in Ref. [1] for the two dimensional orthotropic cases to give (prior to wrinkling),

$$N_{x0} = A\epsilon_{x0} + B\kappa_{x0} \qquad M_{x0} = B\epsilon_{x0} + D\kappa_{x0} \tag{1}$$

with

$$\epsilon_{x0} = \frac{\partial u_0{}^F}{\partial x} \quad \text{and} \quad \kappa_{x0} = -\frac{\partial^2 w_0{}^F}{\partial x^2}. \tag{2}$$

But since we have taken $w_0{}^F = 0$ it follows that $\kappa_{x0} = 0$ with the subsequent simplification of eqn (1).

In the wrinkled condition, the stress-strain, strain-displacement and equilibrium equations for the orthotropic core ($\sigma_{x1} = 0$) can be written,

$$\sigma_{z1} = C\epsilon_{z1}; \qquad \tau_{xz1} = L_{xz}\gamma_{xz1} \tag{3}$$

$$\epsilon_{z1} = \frac{\partial w_1}{\partial z}; \qquad \gamma_{xz1} = \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \tag{4}$$

$$\frac{\partial \tau_{xz1}}{\partial z} = 0; \qquad \frac{\partial \tau_{xz1}}{\partial x} + \frac{\partial \sigma_{z1}}{\partial z} = 0 \tag{5}$$

whilst for the compression face we have,

$$(N_{x0} + N_{x1}) = A(\epsilon_{x0} + \epsilon_{x1}) + B\kappa_{x1} \tag{6}$$

$$(M_{x0} + M_{x1}) = B(\epsilon_{x0} + \epsilon_{x1}) + D\kappa_{x1} \tag{7}$$

$$\epsilon_{x0} + \epsilon_{x1} = \frac{\partial}{\partial x}(u_0{}^F + u_1{}^F); \qquad \kappa_{x1} = -\frac{\partial^2 w_1{}^F}{\partial x^2} \tag{8}$$

$$\frac{\partial}{\partial x}(N_{x0} + N_{x1}) = (\tau_{xz1})_{z=h} \tag{9}$$

$$\frac{\partial^2}{\partial x^2}(M_{x0} + M_{x1}) + (N_{x0} + N_{x1})\frac{\partial^2 w_1{}^F}{\partial x^2} = (\sigma_{z1})_{z=h}. \tag{10}$$

Note that the last two equations represent the equilibrium conditions in the x and z directions

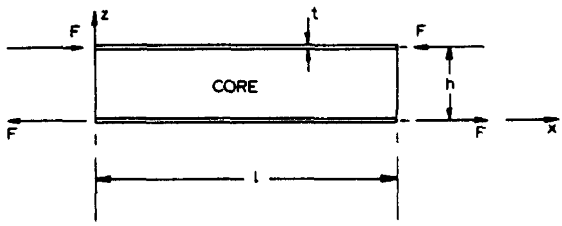

Fig. 1. Beam geometry.

respectively, and that they contain the effects of the stresses from the core acting at the interface. However, in eqn (10) the moment due to the offset core shear stress τ_{xz1} is neglected. This approximation is reasonable providing the faces are thin compared with the core.

We can now manipulate the above face plate equations to give a set of equations in terms of the small changes which occur during wrinkling. Combining (1) with (6) and (7) and remembering that $\kappa_{x0} = 0$ gives,

$$N_{x1} = A\epsilon_{x1} + B\kappa_{x1}; \qquad M_{x1} = B\epsilon_{x1} + D\kappa_{x1}. \tag{11}$$

Equations (2) and (8) combine to give,

$$\epsilon_{x1} = \frac{\partial u_1{}^F}{\partial x}; \qquad \kappa_{x1} = -\frac{\partial^2 w_1{}^F}{\partial x^2} \tag{12}$$

whilst with $(\partial N_{x0}/\partial x) = (\partial^2 M_{x0}/\partial x^2) = 0$ and N_{x1} small compared with N_{x0}, eqns (9) and (10) reduce to

$$\frac{\partial N_{x1}}{\partial x} = (\tau_{xz1})_{z=h} \tag{13}$$

$$\frac{\partial^2 M_{x1}}{\partial x^2} + N_{x0}\frac{\partial^2 w_1{}^F}{\partial x^2} = (\sigma_{z1})_{z=h}. \tag{14}$$

Equations (3)–(5) for the core and eqns (11)–(14) for the faces form the basic set of equations. They will now be manipulated to form one governing differential equation for the wrinkling problem. Since it is no longer necessary to distinguish between suffix 0 and suffix 1 they will be dropped, except for N_{x0}.

Firstly, because of the simplified form of the orthotropic core equations they can be integrated. Equations (5) then become,

$$\sigma_z = -z\frac{\mathrm{d}\tau_{xz}}{\mathrm{d}x} + K_1 \tag{15}$$

and this can be used in the first of eqns (3) with (4) to give

$$w = -\frac{z^2}{2C}\frac{\mathrm{d}\tau_{xz}}{\mathrm{d}x} + z\frac{K_1}{C} + K_2. \tag{16}$$

Similarly u can be obtained from (3) and (4) to give

$$u = z\frac{\tau_{xz}}{L_{xz}} + \frac{z^3}{6C}\frac{\mathrm{d}^2\tau_{xz}}{\mathrm{d}x^2} - \frac{z^2}{2C}\frac{\mathrm{d}K_1}{\mathrm{d}x} - z\frac{\mathrm{d}K_2}{\mathrm{d}x} + K_3. \tag{17}$$

It is now assumed that the tension face remains unstrained during wrinkling so that the functions K_2 and K_3 are both zero since $u = w = 0$ when $z = 0$. Equations (16) and (17) then give the variations of w and u in the core and they can be used for the compressed face plate when $z = h$.

Equations (11) and (12) are now used in the face equilibrium eqns (13) and (14) to give,

$$A\frac{\partial^2 u^F}{\partial x^2} - B\frac{\partial^3 w^F}{\partial x^3} = (\tau_{xz})_{z=h} \tag{18}$$

and

$$B\frac{\partial^3 u^F}{\partial x^3} - D\frac{\partial^4 w^F}{\partial x^4} + N_{x0}\frac{\partial^2 w^F}{\partial x^2} = (\sigma_z)_{z=h}. \tag{19}$$

The condition of displacement compatibility between core and face plate is used when eqns (16) and (17) are substituted into eqns (18) and (19) with eqn (15). This yields two simultaneous ordinary differential equations in K_1 and $(\tau_{xz})_{z=h}$. Then by suitable differentiation and substitution we finally obtain the following governing eighth order equation for $(\tau_{xz})_{z=h}$.

$$q_1\frac{d^8\tau_{xz}}{dx^8} - q_2\frac{d^6\tau_{xz}}{dx^6} + q_3\frac{d^4\tau_{xz}}{dx^4} - q_4\frac{d^2\tau_{xz}}{dx^2} + q_5\tau_{xz} = 0 \tag{20}$$

with

$$\tau_{xz} \equiv (\tau_{xz})_{z=h}$$

and

$$q_1 = \frac{h^3}{6C}\left[\frac{AD - B^2}{Ah + 2B}\right]$$

$$q_2 = \frac{2h}{L_{xz}}\left[\frac{AD - B^2}{Ah + 2B}\right] - \frac{FAh^3}{6C(Ah + 2B)}$$

$$q_3 = \left[\frac{Bh + 2D}{Ah + 2B}\right] + \frac{h}{3}\left[\frac{2Ah + 3B}{Ah + 2B}\right] - \frac{2AhF}{L_{xz}(Ah + 2B)}$$

$$q_4 = \frac{2AC}{L_{xz}(Ah + 2B)} - \frac{2F}{Ah + 2B}; \qquad g_5 = \frac{2C}{h(Ah + 2B)}$$

and where the load N_{x0} has been replaced by $-F$ (Fig. 1).

This equation is conveniently solved by assuming that w_1^F, for the wrinkled face, varies as $a_n \sin(n\pi x/l)$ from which it follows from eqn (18) that $(\tau_{xz})_{z=h}$ must have the form $b_n \cos(n\pi x/l)$. Using this in eqn (20), we find that the condition for wrinkling is given by,

$$q_1\left(\frac{n\pi}{l}\right)^8 + q_2\left(\frac{n\pi}{l}\right)^6 + q_3\left(\frac{n\pi}{l}\right)^4 + q_4\left(\frac{n\pi}{l}\right)^2 + q_5 = 0 \tag{21}$$

for $n = 1, 2$ or $3\ldots$ in turn.

3. NUMERICAL RESULTS

A computer programme was written to evaluate F from eqn (21) for various values of n. Figure 2 shows typical curves for a sandwich beam with cross-ply carbon fibre reinforced plastic (CFRP) faces of total thickness $t = 0.3$ mm and core thickness $h = 25$ mm. The values for the mechanical properties used in the analysis are shown in the figure. The effect of including the adhesive layer is to increase the critical loads as would be expected, and minimum values are reached when n is in the region of 75. This corresponds to a half wave length of wrinkle = 5.33 mm which, in some cases, may be close to the cell size of the honeycomb core. This raises the question of how valid it is under these circumstances to replace the core by a continuum. However, bearing in mind the staggered geometry of the cells, then the unsupported part of the face is not continuous across the width of the beam. Since the buckled wave-form is

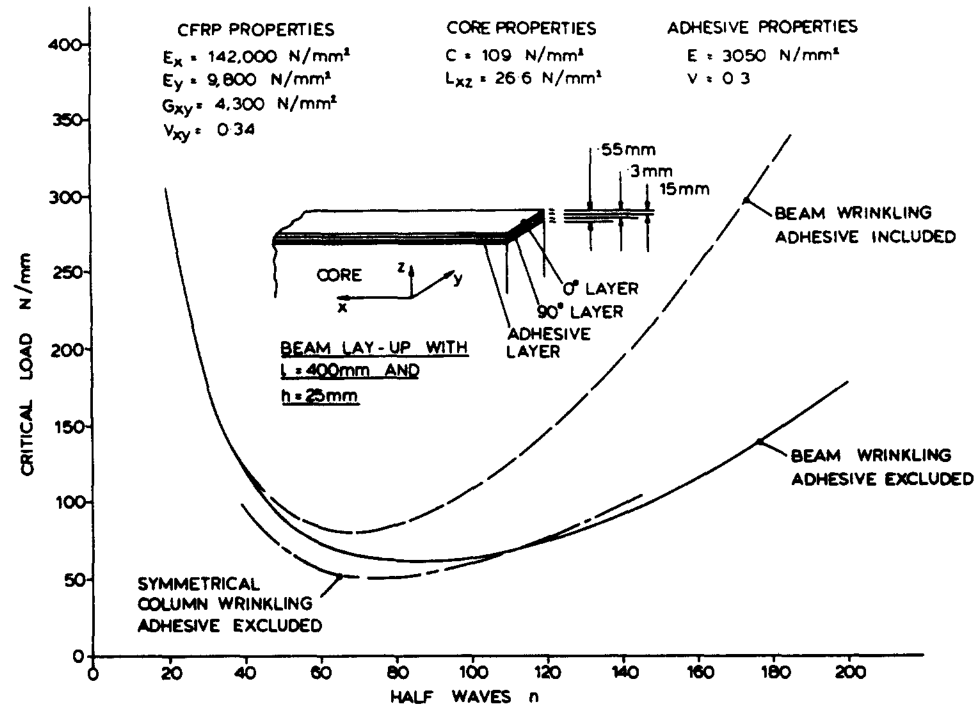

Fig. 2. Crical load vs number of half waves.

invariant in the width direction it follows that some degree of support will be maintained within the wrinkle for all possible wavelengths. This assumes, of course, that there exists a number of cells across the beam width. Nevertheless, when the buckled wave-length is found to be close to the cell size, it would be advisable to check for intra-cellular buckling (dimpling).

It is interesting to compare the above results with the column buckling analysis in Ref. [1] (Fig. 2). To make a direct comparison, the critical column end load has to be halved since it is divided equally between the two faces. The resulting symmetrical wrinkling loads are shown in Fig. 2 and it is seen that they follow the general pattern for the beam but with a lower minimum. Even so, it appears that for this particular example the column analysis gives a good approximation to the critical beam wrinkling load.

Minimum wrinkling loads were obtained for a wide range of face plate thicknesses and core thicknesses for the same 0°/90° lay-up and mechanical properties as given in Fig. 2. The results are presented in Fig. 3 where it can be seen that increasing the face thickness leads to an increase in the wrinkling force as expected. On the other hand, an increase in core thickness has the opposite effect due to the decrease in its support flexibility. The dotted lines show the effect of including the adhesive layer in the analysis which is significant when the faces are very thin.

Two approximate methods of solving the flexural wrinkling problem for isotropic faces are now compared with the present solution. The first takes the Timoshenko[3] solution for a strut on a Winkler foundation of modulus $\beta = (2bC/h)$ where the critical buckling load (P) is given by,

$$P = \frac{\pi^2 EI}{l^2}\left[n^2 + \frac{\beta l^4}{n^2\pi^4 EI}\right]. \tag{22}$$

In fact, this same result can be obtained from the column analysis of Ref. [1] eqn (33) where it is noted that when symmetrical wrinkling occurs, the mid-plane of the column remains flat. This mid-plane is then taken as the tension face in the beam case so that the equivalent beam core thickness is half that of the column thickness. The second method used for comparison is given in Ref. [4]. However, in this case the core is isotropic, and although, in the analysis, the core equations are satisfied, only the face equilibrium equation in the direction normal to the face is

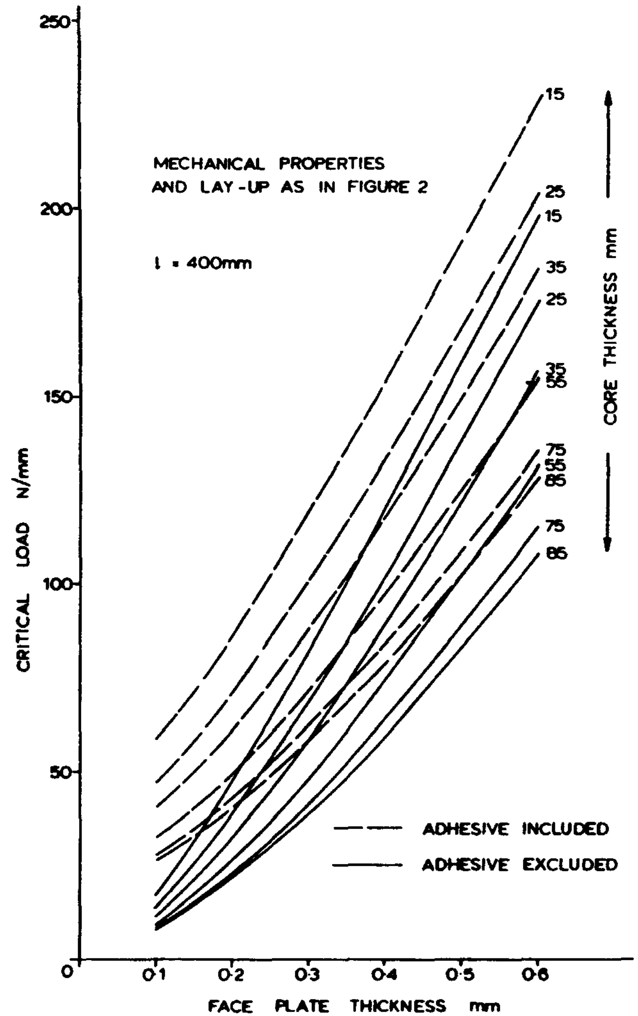

Fig. 3. Variation of beam wrinkling load with face plate thickness and core thickness.

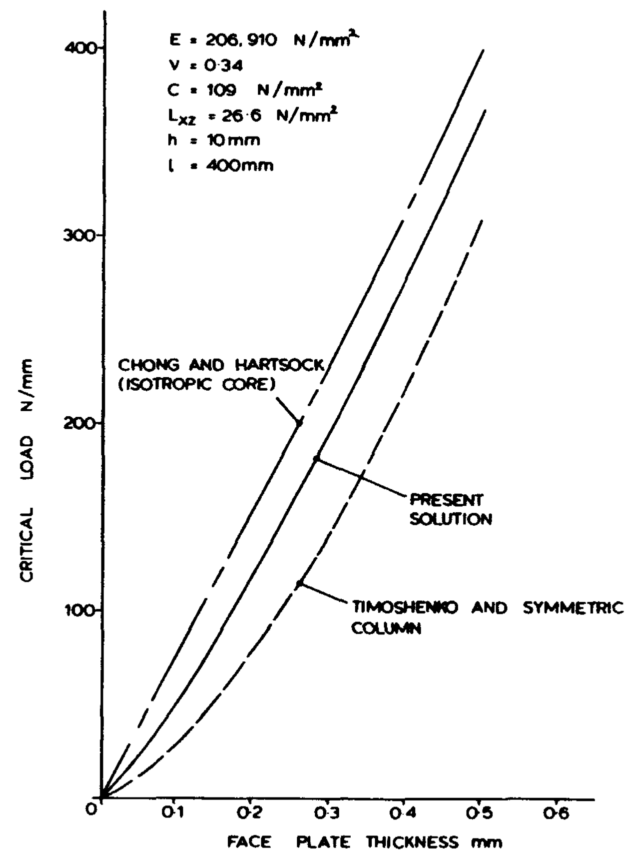

Fig. 4. Comparison with other approximate solutions.

used; the longitudinal equation (eqn 9) is not taken into account. Figure 4 shows the critical load versus face plate thickness for each of the solutions, for the case of an isotropic face. The mechanical constants used in the analyses are given in the figure. There is considerable difference between them, particularly at low thickness values. Chong and Hartsock's isotropic core solution over-estimates the critical loads whilst the Timoshenko and symmetrical column solution underestimates the loads. This latter solution does not take into account the effect of shear stiffness in the core and so one would expect it to be lower than the exact solution. On the other hand, it is not possible to say whether the disagreement with Chong and Hartsock's solution is due mainly to the omission of eqn (9) or to the difference in the type of core. Most certainly both aspects must contribute and the results for this particular example show that it would not be wise to design honeycomb beams on the basis of existing theories for beams with isotropic cores.

4. CONCLUSIONS

A theoretical solution has been given for flexural wrinkling of honeycomb sandwich beams. The governing elasticity equations for the core and laminated face plates have been satisfied, with the assumption that the face plates remain straight before wrinkling and that the associated core stresses are negligible. The effect of the adhesive film bonding the faces to the core has been included and this was found to be important for very thin faces.

Numerical results were compared with results obtained from two approximate solutions, one of which was found to over-estimate the wrinkling load and the other to under-estimate the load. The largest percentage differences were found to occur when the face plates were very thin.

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